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2002 J. Phys. A: Math. Gen. 35 8965

(http://iopscience.iop.org/0305-4470/35/42/308)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **35** (2002) 8965–8967 PII: S0305-4470(02)37640-6

REPLY

Reply to 'Comment on "Self-dressing and radiation reaction in classical electrodynamics"'

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Received 4 July 2002, in final form 14 August 2002
Published 8 October 2002
Online at stacks.iop.org/JPhysA/35/8965

Abstract
It is acknowledged that the expression obtained by Hnizdo (Hnizdo V 1999 *Phys. Rev.* A **60** 4191) in integral form for the time-averaged radiation reaction force $\bar{F}_{RR}$ is correct. It is shown that our expression (Compagno G and Persico F 1999 *Phys. Rev.* A **60** 4196) for the same quantity is also correct, contrary to Hnizdo's claim.

PACS numbers: 03.50.De, 03.70.+k, 12.20.−m

In his comment Hnizdo (H) evaluates the average value $\bar{F}_{RR}$, over a finite time $T = t_1'' - t_1'$, of the radiation reaction force $F_{RR}(t)$ of a nonrotating free charge uniformly and rigidly distributed within a sphere of radius a using expression (3.8) in our paper [1] for a charge initially deprived of the transverse field. H then puts $\bar{F}_{RR}$ in the form

$$\bar{F}_{RR} = \frac{q^2}{T}\int_{t_1'}^{t_1''} \mathrm{d}t_1\, Q(t_1) f(t_1) \tag{1}$$

where $Q(t)$ is the one-dimensional trajectory of the charge, subject to the conditions of slow, small amplitude motion. In this way Hnizdo obtains expression (10) for $f(t_1)$ in the preceding comment and remarks that this expression is the same as that obtained by himself in a previous paper [2],

$$f(t_1) = \frac{1}{2a^3}(\chi - 2)(2 - 2\chi - \chi^2)\Theta(2 - \chi) \qquad \chi = \frac{t_1'' - t_1}{a}. \tag{2}$$

The latter expression for $f(t_1)$ had been criticized by us as incorrect in a paper [3], where instead we fostered use of the following form for $f(t_1)$:

$$f(t_1) = \frac{2}{3}\varrho^2 \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}\frac{1}{c^{n+2}}[\delta^{(n+1)}(t_1'' - t_1) - \delta^{(n+1)}(t_1' - t_1)]\langle r^{n-1}\rangle \tag{3}$$

where $\delta^{(n)} \equiv \mathrm{d}^n\delta/\mathrm{d}t_1^n$; $\langle r^n\rangle = \int_V \mathrm{d}^3\mathbf{x}_1 \int_V \mathrm{d}^3\mathbf{x}_2\, r^n$, $r = |\mathbf{x}_1 - \mathbf{x}_2|$; $V = 4\pi a^3/3$ and ϱ is the uniform charge density within the sphere. It should be noted that there is an obvious sign misprint in expressions (5) and (9) of [3]. Since expression (3) is apparently very different from H's expression (10) in his comment, he deduces that our expression (3) is in error and our criticism of (2) was unjustified.

The following remarks are in order:

1. It is true that in our paper [3] we claimed that H's expression (2) for $f(t_1)$ is incorrect. We are glad to acknowledge that H has now shown that this is not the case.
2. However, we shall show that our expression (3) is also correct and equivalent to (2). In fact using result (4.7) of a paper by Barton [4] to evaluate the factors $\langle r^n \rangle$, expression (3) can be transformed as

$$\begin{aligned} f(t_1) &= \frac{2}{3}\varrho^2 4\pi \frac{V}{c^2} \int_0^{2a} r \left[1 - \frac{3}{2}\frac{r}{2a} + \frac{1}{2}\left(\frac{r}{2a}\right)^3 \right] \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}\frac{r^n}{c^n} \\ &\quad \times \frac{\mathrm{d}}{\mathrm{d}t_1}[\delta^{(n)}(t_1'' - t_1) - \delta^{(n)}(t_1' - t_1)]\,\mathrm{d}r \\ &= \frac{2}{3}\varrho^2 4\pi \frac{V}{c^2} \int_0^{2a} r \left[1 - \frac{3}{2}\frac{r}{2a} + \frac{1}{2}\left(\frac{r}{2a}\right)^3 \right] \\ &\quad \times \left[\delta^{(1)}\left(t_1'' - t_1 - \frac{r}{c}\right) - \delta^{(1)}\left(t_1' - t_1 - \frac{r}{c}\right) \right] \mathrm{d}r. \end{aligned} \tag{4}$$

   The δ-functions in the last line of equation (4) satisfy $\mathrm{d}\delta/\mathrm{d}t_1 \equiv c\,\mathrm{d}\delta/\mathrm{d}r$. Using this property, defining

$$g(r) = r\left[1 - \frac{3}{2}\left(\frac{r}{2a}\right) + \frac{1}{2}\left(\frac{r}{2a}\right)^3\right] \tag{5}$$

   and integrating by parts we obtain

$$f(t_1) = -\frac{8\pi}{3}\varrho^2 \frac{V}{c} \int_0^{2a} \mathrm{d}r\, g'(r) \left[\delta\left(t_1'' - t_1 - \frac{r}{c}\right) - \delta\left(t_1' - t_1 - \frac{r}{c}\right) \right] \tag{6}$$

   where $g' = \mathrm{d}g/\mathrm{d}r$ and we have used $g(0) = g(2a) = 0$. δ-functions integrated in a limited interval satisfy

$$\int_a^b f(x)\delta(x - x_0)\,\mathrm{d}x = f(x_0)\Theta(b - x_0)\Theta(x_0 - a). \tag{7}$$

   Using equation (7) it is easy to see that performing integration the second δ-function in equation (6) does not contribute and we obtain

$$\begin{aligned} f(t_1) &= -\frac{8\pi}{3}\varrho^2 V g'[c(t_1'' - t_1)]\,\Theta\left(\frac{2a}{c} - t_1'' + t_1\right) \\ &= -\frac{8\pi}{3}\varrho^2 V \left\{ 1 - 3\frac{c(t_1'' - t_1)}{2a} + 2\left[\frac{c(t_1'' - t_1)}{2a}\right]^3 \right\} \Theta\left(\frac{2a}{c} - t_1'' + t_1\right). \end{aligned} \tag{8}$$

   The last form of expression (8) can be easily shown to coincide exactly with (2), when the units of measure used by H are taken into account.
3. We never objected to the use of (1) to evaluate $\bar{F}_{RR}$. What we objected to was the procedure adopted by H in his paper [2] (from equations (10) to (17)) in order to perform the integration in (1) approximately. Our expression (3) is particularly convenient in order to evaluate the integral in (1) exactly, as we have done in [3], apart from a sign misprint.

Acknowledgments

This work was supported by the European Union under contract no HPHA-CT-2001-40002 and in part by Comitato Regionale di Ricerche Nucleari e di Struttura della Materia and by Ministero dell'Universitá e della Ricerca Scientifica e Tecnologica. The authors are grateful to H Hnizdo for comunication of results prior to publication.

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